Indian Statistical Institute
Mid-Semestral supplementary Examination
Algebra I
2017-2018

Max Marks: 100 Time: 3 hours.

Answer all questions.

1. Explain why the following statements are true.
(a) If $G/Z(G)$ is cyclic, then G is abelian.
(b) Every characteristic subgroup is normal.
(c) Every abelian group of order pq, where p, q are distinct primes, is cyclic.
(d) $Inn(S_n)$, the group of inner automorphisms of S_n, is isomorphic to S_n.
(e) $Z(S_n) = 1$ for all $n \geq 3$. [6×5]

2. Let G be a group and N a normal subgroup of G. Show that there exists a bijective correspondence between subgroups of G containing N and subgroups of G/N. [10]

3. Let n be a positive integer. Describe the group $Aut(\mathbb{Z}_n)$, where $\mathbb{Z}_n$ is the cyclic group of order n. [12]

4. Prove that if G is a finite group, and p is the smallest prime dividing the order of G, then any subgroup of index p is normal. [8+10]

5. (a) Deduce *Class Equation.*
(b) Show that a group of order p^n, where p is a prime and $n \geq 1$, has a non-trivial center. [10+8]

6. Show that the order of the centraliser $C_{S_n}((12)(34))$ is $(n-4)! \times 8$, for all $n \geq 4$. Determine the elements explicitly. [12]